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Resistance of a Fast, Round Bilge Hull in Shallow Water

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Existing linearized wave theory for the resistance of a ship in both deep and shallow water has been used to calculate the effect of shallow water at high subcritical and supercritical speeds. The ship was represented by a simplified hull shape which was symmetrical fore and aft, with parabolic waterlines and a rectangular cross section. The calculations have been compared with the experimental results of Sturtzel and Graff for a family of five round bilge displacement hulls. The comparison between the simplified theory and the experimental data showed reasonable agreement. The largest differences occurred near the resistance peak at high subcritical Froude numbers and were thought to be primarily a result of the simplified hull shape assumed in the theoretical work, although other factors are discussed.

Nomenclature

C_B	= block coefficient of hull, ∇/LBT
C_r	= residual resistance coefficient, $R_r/\frac{1}{2}\rho V^2 S$
C_w	= wave resistance coefficient, $R_w/\frac{1}{2}\rho V^2 S$
F	= Froude number of ship, $V/(gL)^{1/2}$
F_{nh}	= depth Froude number, $V/(gH)^{1/2}$
$f(x,z)$	= equation of ship's wetted surface
g	= acceleration due to gravity
H	= water depth
I,J	= integral over the wetted surface of the ship
L	= length of ship
R_r	= residual resistance of ship
R_w	= wave resistance of ship
R_w^*	= dimensionless wave resistance of ship, $R_w/8\rho g B^2 T^2/\pi L$
S	= wetted surface area of ship
T	= draught of ship
V	= velocity of ship
x,y,z	= coordinates in horizontal, transverse, and vertical directions
γ	= integration variable, $\gamma_0\lambda$
γ_0	= $1/2F^2$
ζ	= dimensionless coordinate, z/T
η	= dimensionless coordinate, $f(x,z)/(B/2)$
θ	= $2T/L\gamma^2/\gamma_0$
λ	= integration variable
ξ	= dimensionless coordinate, $x/(L/2)$
ρ	= water density
∇	= volume displacement of hull

Suffixes

h	= finite water depth
∞	= infinite water depth

Introduction

RECENT trends in ship size and the increased congestion of the shipping routes has led to a corresponding interest in the effects of shallow water on ships. However, most of the associated research refers to large ships moving at medium or slow speeds, particularly in shallow water, so that in terms of the critical wave speed these are subcritical conditions, i.e.,

$$V < (gH)^{1/2} \tag{1}$$

where V is the ship's speed, H the water depth, and g the gravitational acceleration. At such subcritical speeds the effect of shallow water is to increase the ship's resistance compared with the deep water value but at supercritical speeds a totally different effect has been shown to occur, with the result that the resistance can actually be lower than the deep water value. As many smaller ships, such as those used for fast patrol boats, can achieve relatively high speeds and may also need to operate in shallow water, it is an advantage to be able to predict the changes in resistance which will occur in shallow water for the high subcritical and supercritical speed ranges.

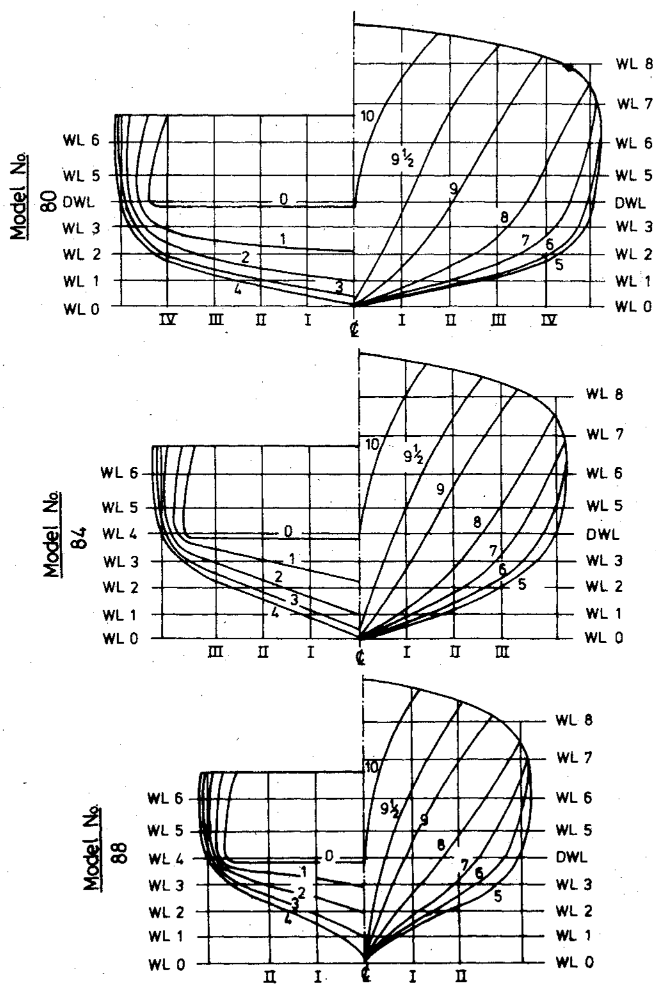

Fig. 1 Body lines of Models 80, 84, and 88 round bilge displacement hulls.

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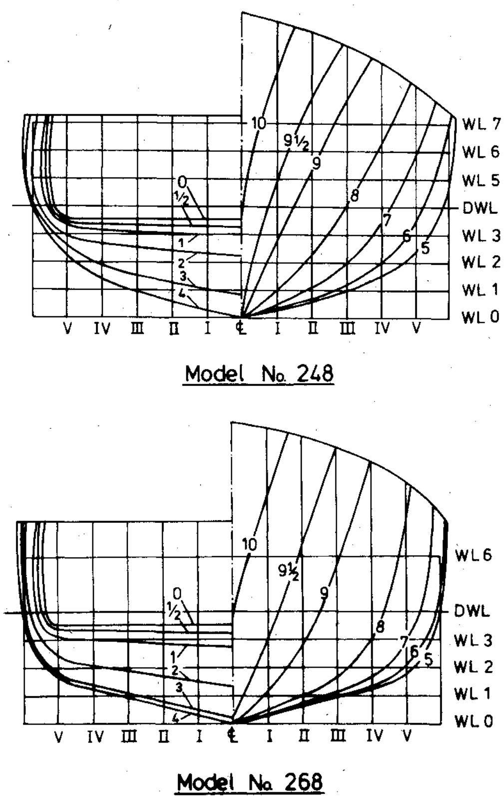

Fig. 2 Body lines of Models 248 and 268 round bilge displacement hulls.

This paper has used existing linearized wave theory in order to make a comparison with experimental data obtained by Sturtzel and Graff[1,2] for a family of five related round bilge displacement hulls. The five hull models all had the same waterline length and draught but had variations in beam and hull shape as can be seen from the diagrams in Figs. 1 and 2 and the data given in Table 1.

In the theoretical work a simplified hull was used as an initial approximation to the real hull shapes. This simplified hull, which had the same waterline length and draught as the real hulls, consisted of parabolic waterlines and rectangular cross sections as shown in Fig. 3.

The purpose of the comparison was to determine how well the theory, using this simplified hull shape, modelled the effect of shallow water on the real hulls.

Wave Resistance Theory

The method of calculating the wave resistance is based on the linearized theory of ship resistance given by Michell[3] for deep water of unrestricted width and by Srettensky[4] for shallow water. The effects of shallow water were calculated by Kirsch,[5] where the ship's hull was approximated by a symmetrical fore-and-aft shape with parabolic waterlines and rectangular cross sections, but the work did not include any comparison with experimental data. In the present work a similar procedure has been followed but the equations have been manipulated so that the nondimensional resistance R^* is independent of beam B and depends only on the length/draught ratio (L/T) of the hull, whereas Kirsch had a dependence both on length/beam ratio (L/B) and beam/draught ratio (B/T). As a result the ratio of the nondimensional resistance in shallow and deep water is also not dependent on B and the calculations for the one simplified hull could therefore be compared with the experimental data for the five real hulls since they all had the same length and draught. The equations are given below and calculations were made using these formulas to enable a comparison with experimental data to be made.

Table 1 Model principal dimensions

Item	Theoretical hull form	Experimental models				
		80	84	88	248	268
L,m	2.5	2.5	2.5	2.5	2.5	2.5
L/B	–	5.05	6.06	7.58	6.06	6.06
T,m	0.11	0.11	0.11	0.11	0.11	0.11
S,m^2	–	1.224	0.994	0.815	0.972	1.065
∇, m^3	–	0.06803	0.04522	0.02714	0.045	0.057
C_B	–	0.500	0.399	0.299	0.397	0.502

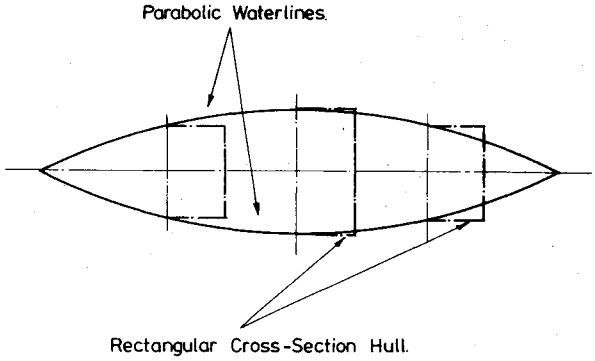

Fig. 3 Theoretical model hull shape.

In deep water of unrestricted width the nondimensional resistance is

$$R^*_{w\infty} = \int_{\gamma}^{\infty} \frac{(\gamma/\gamma_0)^2}{[(\gamma/\gamma_0)^2 - 1]^{1/2}} [I_\infty^2 + J_\infty^2] \mathrm{d}\gamma \tag{2}$$

where

$$I_\infty = \int \int_0^1 \frac{\partial \eta}{\partial \xi} (\xi,\zeta) e^{-\theta\zeta} \cos(\gamma\xi) \mathrm{d}\xi \mathrm{d}\zeta$$

and

$$J_\infty = \int \int_0^1 \frac{\partial \eta}{\partial \xi} (\xi,\zeta) e^{-\theta\zeta} \sin(\gamma\xi) \mathrm{d}\xi \mathrm{d}\zeta$$

The corresponding equation for the nondimensional resistance in shallow water is

$$R^*_{wh} = \frac{L}{4} \int_{\bar\gamma_0}^{\infty} \frac{(I_h^2 + J_h^2)}{[\bar\gamma^2 - (\bar\gamma/F^2L)\tanh(\bar\gamma H)]^{1/2}} \frac{\bar\gamma \mathrm{d}\bar\gamma}{\cosh^2(\bar\gamma H)} \tag{3}$$

where

$$I_h = \int \int_0^1 \frac{\partial \eta}{\partial \xi} (\xi,\zeta) \cosh[\bar\gamma(H - T\zeta)] \times \cos\left[\frac{L}{2} \xi \left(\frac{\bar\gamma}{F^2L} \tanh(\bar\gamma H)\right)^{1/2}\right] \mathrm{d}\xi \mathrm{d}\zeta$$

and

$$J_h = \int \int_0^1 \frac{\partial \eta}{\partial \xi} (\xi,\zeta) \cosh[\bar\gamma(H - T\zeta)] \times \sin\left[\frac{L}{2} \xi \left(\frac{\bar\gamma}{F^2L} \tanh(\bar\gamma H)\right)^{1/2}\right] \mathrm{d}\xi \mathrm{d}\zeta$$

For $V < (gH)^{1/2}$, $\bar\gamma_0$ is obtained from the equation

$$\tanh(\bar\gamma_0 H) = (V^2/gH)\bar\gamma_0 H \tag{4}$$

and for $V > (gH)^{1/2}$, $\bar\gamma_0 = 0$.

The simplified hull was assumed to be symmetrical about the midship section with the result that the expressions I_∞ and I_h are both equal to zero. Furthermore, by also assuming that the simplified hull consisted of parabolic waterlines and rectangular cross sections, the equation for the surface of the hull can be separated into two factors, one a function of length only and the other a function of draught only. The equation for this simplified hull is

$$\eta = 1 - \xi^2 \tag{5}$$

Equations (2) and (3) can therefore be written as

$$R^*_{w\infty} = \int_\gamma^\infty \frac{(\gamma/\gamma_0)^2}{[(\gamma/\gamma_0)^2 - 1]^{1/2}} J^2_\infty \mathrm{d}\gamma \tag{6}$$

where

$$J_\infty = \int \int_0^1 \frac{\partial \eta}{\partial \xi} e^{-\theta \zeta} \sin(\gamma \xi)\, \mathrm{d}\xi \mathrm{d}\zeta$$

and for shallow water

$$R^*_{wh} = \frac{L}{4} \int_{\bar{\gamma}_0}^\infty \frac{J^2_h}{[\bar{\gamma}^2 - (\bar{\gamma}/F^2 L)\tanh(\bar{\gamma} H)]^{1/2}} \frac{\bar{\gamma}\mathrm{d}\bar{\gamma}}{\cosh^2(\bar{\gamma} H)} \tag{7}$$

with

$$J_h = \int \int_0^1 \frac{\partial \eta}{\partial \xi} \cosh[\bar{\gamma}(H - T\zeta)]$$

$$\times \sin\left[\frac{L}{2}\xi\left(\frac{\bar{\gamma}}{F^2 L}\tanh(\bar{\gamma} H)\right)^{1/2}\right] \mathrm{d}\xi \mathrm{d}\zeta$$

It can be seen that after substituting the equation for the simplified hull [Eq. (5)] in the expressions for J_∞ and J_h these can be integrated analytically. The final integrations to obtain values for the nondimensional wave resistances in deep and shallow water at each ship speed, $R^*_{w\infty}$ and R^*_{wh}, were carried out numerically.

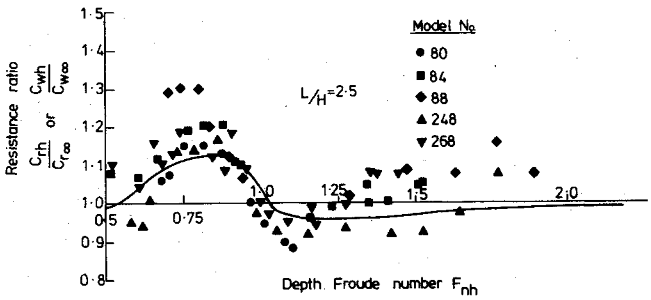

Fig. 4 Comparison between theory and experiment for water depth ratio $L/H = 2.5$.

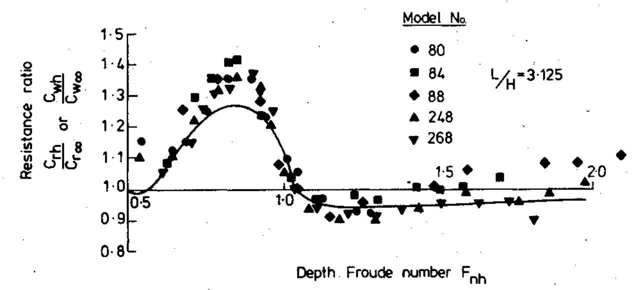

Fig. 5 Comparison between theory and experiment for water depth ratio $L/H = 3.125$.

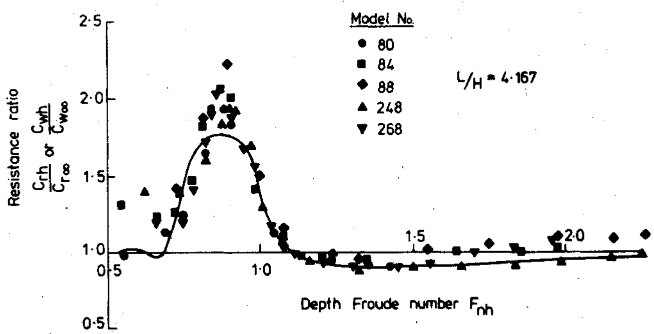

Fig. 6 Comparison between theory and experiment for water depth ratio $L/H = 4.167$.

Discussion

The source of data for comparison with the theoretical calculations was the results obtained by Sturtzel and Graff[1,2] for a series of round bilge fast displacement hulls with the same overall length and maximum draught but modified hull lines. The experimental results for the five hulls for which a complete set of data was available were manipulated to give the ratio of residual resistance in shallow water to residual resistance in deep water for the same ship speed by subtracting the calculated frictional resistance, using the International Towing Tank Conference formula,[6] from the measured total resistance. This residual resistance ratio was then plotted against Froude number based on the ship speed and the depth of water for comparison with the theoretical wave resistance ratio for shallow and deep water calculated for the simplified hull with the same waterline length and maximum draught.

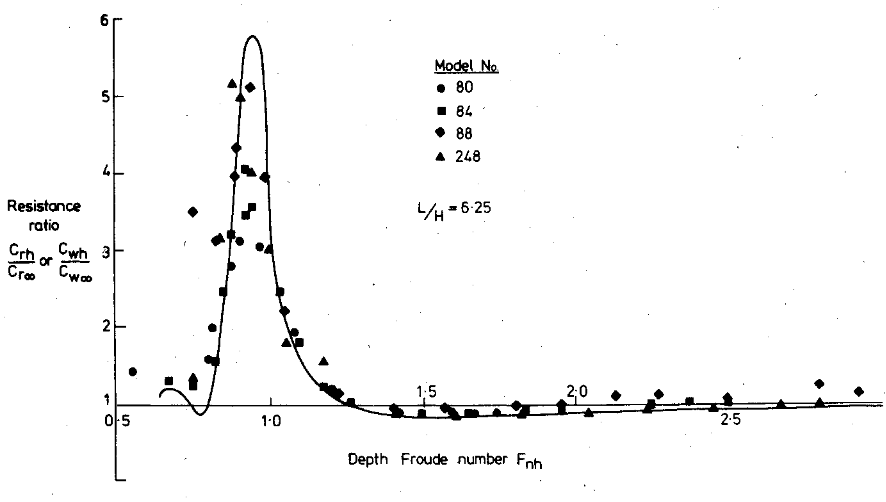

Fig. 7 Comparison between theory and experiment for water depth ratio $L/H = 6.25$.

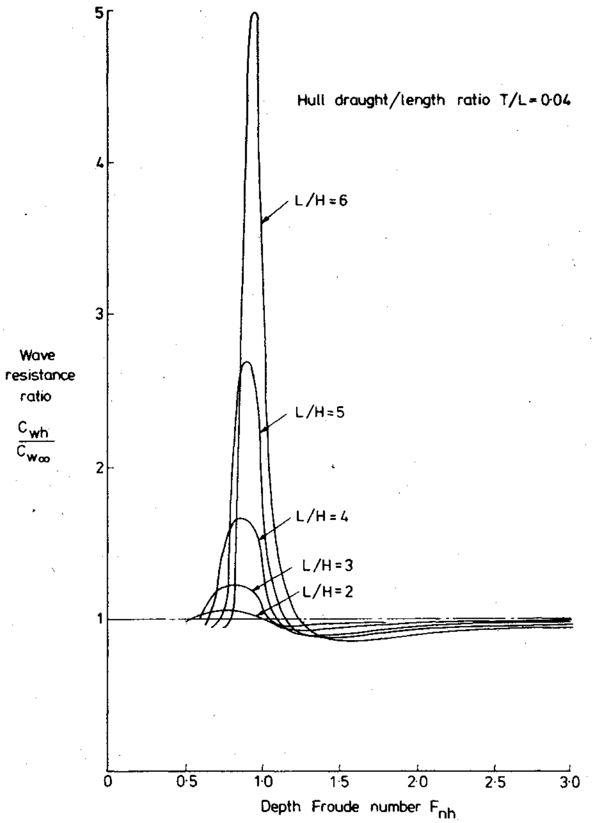

Fig. 8 Shallow water effect on the theoretical wave resistance of a simplified hull.

The theoretical curves and the experimental data for the five hulls are compared in Figs. 4-7 for the four different water depths giving hull length/water depth ratios (L/H) of 2.5, 3.125, 4.167, and 6.25 for increasingly shallow water. The trend of the curves shows general agreement between theory and experiment, indicating an increase in resistance ratio at high subcritical speeds and a reduction in resistance ratio at supercritical speeds. The curves also show that both the increase in resistance at subcritical speeds and the reduction in resistance at the supercritical speeds become larger as the water becomes shallower, i.e., as L/H becomes larger.

It was noted for the three deeper depths of water (Figs. 4-6) the experimental values were higher than the theoretical curve in the region of the subcritical resistance peak, although occurring over substantially the same range of Froude numbers. For the shallowest water depth (Fig. 7) the experimental values appear to be lower than the theoretical curve, although the differences are quite large between the four models for which the data were measured.

At supercritical speeds the average of the experimental values agrees reasonably well with the theoretical curve. It is suggested that the wide scatter of the experimental values between the various models is partially due to the differing effects of trim between deep and shallow water since each hull was free to trim.

Therefore overall the comparison has shown that the theoretical model of the effect of shallow water on the resistance of a ship shape is in reasonable agreement with the experimental data for a series of five round bilge fast displacement hulls, predicting both the increase in resistance that can be expected at subcritical Froude numbers and the reduction in resistance at supercritical Froude numbers. It can be seen in Fig. 8, which shows the theoretical curves for a range of ratios of hull length to water depth, that the effect of the shallow water becomes greater as the water gets shallower for a particular hull, that is, as the hull length/water depth ratio increases. In the subcritical range the peak of the curves, showing the maximum increase in resistance due to shallow water, moves closer to the critical Froude number as the hull length/water depth ratio is increased, while in the supercritical range the position of the maximum reduction in resistance moves further away from the critical Froude number.

In assessing the overall agreement between the theoretical prediction and the experimental data and therefore in considering the use of the theory as a method of predicting the effect of shallow water, a number of factors may have affected the comparison and are listed below.

Experimental Data

1) The experimental values are the ratio of residual resistance in shallow and deep water obtained by deducting a value for the frictional resistance from the measured model resistance as explained in item 2. The residual resistances will therefore include other resistance contributions and are not strictly comparable with the theoretical wave resistance.

2) The frictional resistance was calculated using the International Towing Tank Conference formula.[6] This formula has been developed on the assumption that both the water depth and width are effectively unlimited and is therefore likely to become less accurate as the water depth decreases since there will be an interaction between the hull and the bottom which may alter the flowfield and hence affect the value of the frictional resistance. It is also really a friction correlation formula rather than a true calculation of the frictional resistance.

3) The frictional resistance was calculated using the static wetted area in the absence of more detailed information. The actual wetted area may be significantly different, particularly at speeds close to the critical wave speed in shallow water.

Theoretical Method

1) The theoretical model assumed a simplified hull shape which was symmetrical fore and aft about the midship section, with waterlines which were parabolic and cross sections which were rectangular. This simple hull can therefore be considered only as a first approximation to a real hull shape.

2) No attempt was made to include the effect of a change in trim on the resistance of the hull resulting from the effects of varying buoyancy or dynamic forces. The experimental data showed that on the model hull there were sometimes significant changes in trim between the deep and shallow water tests particularly close to the critical Froude number.

Conclusions

A comparison between the theoretical prediction of the effect of shallow water on ship resistance, based on existing linearized wave resistance theory, and data for a family of five round bilge displacement hulls has shown reasonable agreement. The largest differences occurred near the resistance peak at high subcritical Froude numbers where the theory gave lower values. The differences between the theoretical model and the experimental data are likely to be mainly due to the simplified hull form used in the theory and further work will be aimed at improving the theoretical representation of the real hull shape.

Both theory and experiment show that the effect of shallow water is to increase the ship's resistance at subcritical speeds but to decrease the resistance at supercritical speeds—both effects becoming larger as the ratio of hull length to water depth is increased.

It is suggested that the theory can be used as a first approximation to determine the effects of shallow water on a fast displacement hull.

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